EVALUATION OF VARIATIONS OF BACKMIXING IN BUBBLE COLUMN TYPE FERMENTORS IN DEPENDENCE ON THE AXIAL COORDINATE

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Analysis of experimental data from a batch column has shown the coefficient of backmixing to increase in dependence on the distance from the column bottom. Limiting values may differ by up to an order of magnitude. Original method has been worked out, based on the method of moments, to process experimental data obtained as a response of the column to a tracer impulse.

This paper is a continuation of the previous communications[1,2] dealing with the problem of the assessment of aeration capacity in bubble column type fermentors. Intensity of mixing and aeration are important parameters which may affect the course of fermentation. From the standpoint of its construction the fermentor of the bubble column type is the simplest bioreactor. In the transfer of the process from a laboratory to industrial scale it is necessary to know well the engineering characteristics of the apparatus. In this context a question arised as to the justification of the generally accepted assumption of constant coefficient of axial mixing in the whole volume of the apparatus.

Deckwer reported[3] a linear dependence of the coefficient of axial mixing on the volume rate of air in small size columns. Considering the fact that in tall apparatuses variations of the hydrostatic pressure occur and hence the volume rate of air varies linearly with the axial coordinate, it may be expected that also the coefficient of axial mixing would vary linearly. In fermentors serving for aerobic treatment of waste waters[4] the local differences in the intensity of mixing may be considerable which in turn may affect the efficiency of the equipment.

THEORETICAL

The non-ideal mixing in apparatuses of the column type under the batch operation may be described by the following dispersion model

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial z}\left(E_z \frac{\partial c}{\partial z}\right) + r(c) \qquad (1)$$

where c designates concentration of the species, E_z is the coefficient of backmixing, z is and axial coordinate and $r(c)$ is the rate of formation of the species.

For the assessment of intensity of mixing systems without chemical reaction are used $(r(c) = 0)$ while the unsteady behaviour is induced by injection of a tracer. The response of the column is then monitored at various time instants and axial positions.

In solving Eq. (*1*) one starts from the assumption of constant E_z which leads to an analytical expression for the transient development of the concentration in time and space[5,6]. Crank[7] and Mingle[8] give solutions with *a priori* given type of concentration dependence of E_z.

Evaluation of the dependence of E_z on the axial coordinate in continuous equipment has been published in refs[9-12].

In this work we have concentrated on the method of evaluation of E_z as it varies along the length of the column under the batchwise operation. Such a method has not been published so far.

Methodically we have utilized the approach published by Blyachman[12], based on the method of moments, to solve Eq. (*1*) under the following initial and boundary conditions

$$\partial c/\partial z = 0 \quad \text{for} \quad \begin{aligned} z &= 0 \\ z &= L \\ t &> 0 \end{aligned} \tag{2a}$$

$$\begin{aligned} c(z, 0) &= 0 \quad \text{for} \quad 0 \leqq z \leqq z_0 \\ c(z, 0) &= c_0 \quad \text{for} \quad z_0 < z \leqq L \end{aligned} \tag{2b}$$

when the layer penetrated by the tracer after its injection on the liquid level extends between the coordinates z_0 and L. The coordinate of the column bottom is 0 and that of the liquid level is L. In the practical case the height of the layer saturated by the tracer is negligible with respect to the column height.

The set of equations (*1*) and (*2*) is rendered dimensionless by the following transformation of the axial coordinate $x = z/L$ and concentration $y = 1 - c/c_E$. Here c_E designates concentration of the tracer after its uniform dispersion in the whole apparatus (cup-mixing mean concentration). The dimensionless relationships then take the form

$$\frac{\partial y}{\partial t} = \frac{\partial}{\partial x}\left(A(x)\frac{\partial y}{\partial x}\right), \tag{3}$$

$$\frac{\partial y}{\partial x} = 0 \qquad \text{for} \quad x = 0$$
$$x = 1 \tag{4a}$$
$$t > 0$$

$$y(x, 0) = 1 \qquad \text{for} \quad x \leqq x_0 \tag{4b}$$
$$y(x, 0) = x_0/(x_0 - 1) \quad \text{for} \quad x > x_0$$

Integration of Eq. (*3*) over the interval $\langle 0, \infty)$ of t then yields the zero-th order moment:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(A(x)\,\frac{\mathrm{d}M(x)}{\mathrm{d}x}\right) = -y(x, 0)\,. \tag{5}$$

Integration of this expression along x yields

$$M(x) = M(0) - \int_0^x \frac{x}{A(x)}\,\mathrm{d}x\,. \tag{6}$$

In order to obtain $M(0)$ a material balance on the tracer is used

$$\int_0^L \frac{\mathrm{d}c}{\mathrm{d}t}\,\mathrm{d}z = 0 \tag{7}$$

modified into the dimensionless form by the above given transformations, multiplication by $t\,\mathrm{d}t$ and integration over the interval $\langle 0, \infty)$ to the form

$$\int_0^1 M(x)\,\mathrm{d}x = 0\,. \tag{8}$$

Applying the balance (*8*) to Eq. (*6*) one obtains

$$M(0) = \int_0^1 \left[\int_0^x \frac{x}{A(x)}\,\mathrm{d}x\right]\mathrm{d}x\,. \tag{9}$$

On starting from the assumption of linear variation of the coefficient of backmixing along the column length in the form $A(x) = ax + b$, and integrating Eq. (*6*) with the aid of Eq. (*9*), we obtain

$$M(x) = 1/2a - b((a + b)\ln(a + b) - a - b\ln(b))/a^3 - x/a + \\ + b(\ln(ax + b))/a^2 \; ; \quad b > 0 \\ a + b > 0 \,. \tag{10}$$

EXPERIMENTAL

The derived relations have been applied to experimental data obtained by measurement in a batch bubble column for aerobic cultivation of yeast 5·5 m high and 185 mm in internal diameter, aerated and mixed by an ejector. The experimental equipment has been described in more detail in the work of Janík[13] and Prokop[14]. The course of mixing was monitored through the perturbation of pH (ref.[15]). The spread of the perturbation was detected at three different heights and for the purpose of demonstration of the method two extreme cases have been selected from the set of experimental data differring by the intensity of aeration[13].

RESULTS AND DISCUSSION

Using the derived relationships, moments of the transient development of the transformed concentration H^+ were computed first. These data are tabulated in Table I. Results of the processing, based on the hypothesis of constant coefficient of backmixing, E_z, for individual experiments, are also shown in Table I, together with the variance of this coefficient. Using the standard recommended method a physically unrealistic value of E_z was obtained for the coordinate $x = 0·515$. From this finding it follows that the standard method does not suit the experimental data even qualitatively.

Starting from the idea of linearly variable E_z along the column length ($A(x) = ax + b$) a nonlinear regression yields values of parameters characterizing the

TABLE I

Moments of the transient development of the transformed concentration H^+. Comparison of constant and variable coefficients of backmixing

Intensity of aeration $m^3\,m^{-3}\,s^{-1}$	x	$M(x)$	E_z^{const} $m^2\,s^{-1}$	$s^2_{E_z}$	$E_z(z)$ $m^2\,s^{-1}$	$s^2_{E_z}(z)$
$1·07 \,.\, 10^{-3}$	0·102	94·25	$3·04 \,.\, 10^{-2}$	0·47	$2·73 \,.\, 10^{-2}$	$5·02 \,.\, 10^{-2}$
	0·458	36·16	$3·02 \,.\, 10^{-2}$	0·47	$2·97 \,.\, 10^{-2}$	$9·68 \,.\, 10^{-2}$
	0·625	−267·90	$1·88 \,.\, 10^{-3}$	0·26	$3·08 \,.\, 10^{-2}$	$1·19 \,.\, 10^{-1}$
$7·48 \,.\, 10^{-3}$	0·084	84·84	$5·02 \,.\, 10^{-2}$	0·43	$1·96 \,.\, 10^{-2}$	$8·68 \,.\, 10^{-3}$
	0·378	63·55	$3·91 \,.\, 10^{-2}$	0·18	$4·24 \,.\, 10^{-2}$	$1·95 \,.\, 10^{-2}$
	0·515	−68·63	$-1·30 \,.\, 10^{-2}$	—	$5·31 \,.\, 10^{-2}$	$2·37 \,.\, 10^{-2}$

variation of E_z, and hence also the variation of mixing, along the column length. Resulting coefficients of backmixing, obtained by this method, as well as the corresponding variances are tabulated in Table I. From the presented data it follows that the intensity of mixing in a column reactor increases in the direction from the air distributor toward the liquid level. Local value of E_z near the liquid level in case of higher intensity of aeration is more than twice the value near the column bottom. The variance of E_z for the case of presumed linear dependence of E_z, as follows from Table I, is by an order of magnitude lower in comparison with the value corresponding to the standard method of evaluation. This finding speaks clearly in favour of the method of evaluation of the intensity of mixing in apparatuses of bubble column type recommended in this work. Our results agree with experimental data of Kölbel and coworkers[16] which measured directly local values of the coefficient of backmixing from the oscillations of gas hold-up.

LIST OF SYMBOLS

$A(x) = E_z/L^2$	transformed coefficient of backmixing
a, b	parameters of linear dependence of $A(x)$ on the longitudinal coordinate
c	concentration of tracer
E_z	coefficient of backmixing of liquid phase
L	column length
$M(x)$	initial moment of tracer concentration
$r(c)$	rate of formation of species
$s^2_{E_z}$	variance of the coefficient of backmixing
t	time
x	dimensionless axial coordinate
$x_0 = z_0/L$	
y	dimensionless tracer concentration
z	axial coordinate
z_0	axial coordinate of the edge of the layer saturated with the tracer at the instant of injection

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